Quadratic Models; Building Quadratic Functions from Data

Example: Maximizing Revenue

In economics, revenue *R*, in dollars, is defined by the amount of money received from the sale of an item and is equal to the selling price *p*, in dollars, of the item times the number *x* of units actually sold. That is, $\boldsymbol{R = xp}$**.**

The price *p* in dollars and the quantity *x* sold of a certain product obey the demand equation

$$p = -\frac{1}{3}x + 100,\ 0 \le x \le 300.$$

a) Express the revenue *R* as a function of *x*.
b) What is the revenue if 100 units are sold?
c) What quantity *x* maximizes revenue? What is the maximum revenue?
d) What price should the company charge to maximize revenue?

Example: Maximizing the Area Enclosed by a Fence

Beth has 3000 feet of fencing available to enclose a rectangular field.
a) Express the area *A* of the rectangle as a function of *x*, where *x* is the length of the rectangle.
b) For what value of *x* is the area largest?
c) What is the maximum area?
d) What are the dimensions of the field with the maximum area?

Example: Analyzing the Motion of a Projectile

A projectile is fired at an inclination of 45° to the horizontal, with a muzzle velocity of 100 feet per second. The height h of the projectile is given by $h(x) = \frac{-32x^2}{(100)^2} + x$, where x is the horizontal distance of the projectile from the firing point.

a) At what horizontal distance from the firing point is the height of the projectile a maximum?

b) Find the maximum height of the projectile.

c) At what horizontal distance from the firing point will the projectile strike the ground?

d) Using a graphing calculator, graph the function h, $0 \le x \le 350$.

e) Using a graphing calculator, verify the results obtained in parts b) and c).

Example: A Parabolic Arch

A parabolic arch has a span of 120 feet and a maximum height of 25 feet. Choose a suitable rectangular coordinate axes and find the equation of the parabola. Then calculate the height of the arch at points 10 feet, 20 feet, and 40 feet from the center.